

11.7 Volume of Pyramids and Cones



Before

You found surface areas of pyramids and cones.

Now

You will find volumes of pyramids and cones.

Why?

So you can find the edge length of a pyramid, as in Example 2.

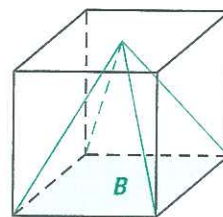
Key Vocabulary

- pyramid
- cone
- volume



CC.9-12.G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*

Recall that the volume of a prism is Bh , where B is the area of a base and h is the height. In the figure at the right, you can see that the volume of a pyramid must be less than the volume of a prism with the same base area and height. As suggested by the Activity, the volume of a pyramid is one third the volume of a prism.



THEOREMS

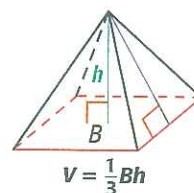
For Your Notebook

THEOREM 11.9 Volume of a Pyramid

The volume V of a pyramid is

$$V = \frac{1}{3}Bh,$$

where B is the area of the base and h is the height.

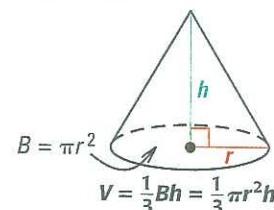


THEOREM 11.10 Volume of a Cone

The volume V of a cone is

$$V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h,$$

where B is the area of the base, h is the height, and r is the radius of the base.



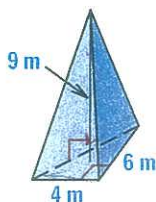
EXAMPLE 1 Find the volume of a solid

Find the volume of the solid.

APPLY FORMULAS

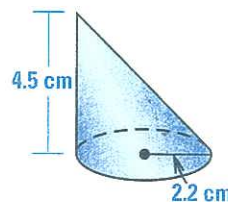
In an *oblique cone*, the vertex is not directly over the center of the base. By Cavalieri's Principle, the volume formula for a right cone works for oblique cones.

a.



$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}\left(\frac{1}{2} \cdot 4 \cdot 6\right)(9) \\ &= 36 \text{ m}^3 \end{aligned}$$

b.



$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}(\pi r^2)h \\ &= \frac{1}{3}(\pi \cdot 2.2^2)(4.5) \\ &= 7.26\pi \\ &\approx 22.81 \text{ cm}^3 \end{aligned}$$

EXAMPLE 2 Use volume of a pyramid

xy ALGEBRA Originally, the pyramid had height 144 meters and volume 2,226,450 cubic meters. Find the side length of the square base.



Khafre's Pyramid, Egy

Solution

$$V = \frac{1}{3}Bh \quad \text{Write formula.}$$

$$2,226,450 = \frac{1}{3}(x^2)(144) \quad \text{Substitute.}$$

$$6,679,350 = 144x^2 \quad \text{Multiply each side by 3.}$$

$$46,384 \approx x^2 \quad \text{Divide each side by 144.}$$

$$215 \approx x \quad \text{Find the positive square root.}$$

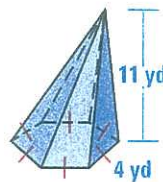
► Originally, the side length of the base was about 215 meters.

**GUIDED PRACTICE** for Examples 1 and 2**APPLY FORMULAS**

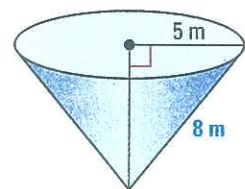
In an *oblique pyramid*, the vertex is not directly over the center of the base. By Cavalieri's Principle, the volume formula for a pyramid works for oblique pyramids.

Find the volume of the solid. Round your answer to two decimal places, if necessary.

1. Hexagonal pyramid



2. Right cone



3. The volume of a right cone is 1350π cubic meters and the radius is 18 meters. Find the height of the cone.

EXAMPLE 3 Use trigonometry to find the volume of a cone

Find the volume of the right cone.

Solution

To find the radius r of the base, use trigonometry.

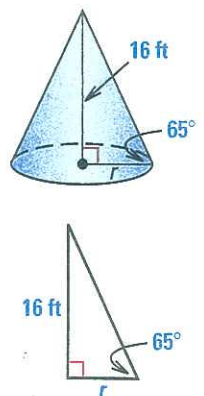
$$\tan 65^\circ = \frac{\text{opp.}}{\text{adj.}} \quad \text{Write ratio.}$$

$$\tan 65^\circ = \frac{16}{r} \quad \text{Substitute.}$$

$$r = \frac{16}{\tan 65^\circ} \approx 7.46 \quad \text{Solve for } r.$$

Use the formula for the volume of a cone.

$$V = \frac{1}{3}(\pi r^2)h \approx \frac{1}{3}\pi(7.46^2)(16) \approx 932.45 \text{ ft}^3$$

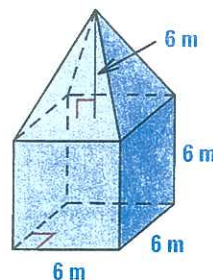


EXAMPLE 4 Find volume of a composite solid

Find the volume of the solid shown.

Solution

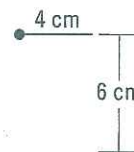
$$\begin{aligned}
 \text{Volume of solid} &= \text{Volume of cube} + \text{Volume of pyramid} \\
 &= s^3 + \frac{1}{3}Bh && \text{Write formulas.} \\
 &= 6^3 + \frac{1}{3}(6)^2 \cdot 6 && \text{Substitute.} \\
 &= 216 + 72 && \text{Simplify.} \\
 &= 288 && \text{Add.}
 \end{aligned}$$



▶ The volume of the solid is 288 cubic meters.

EXAMPLE 5 Solve a multi-step problem

SCIENCE You are using the funnel shown to measure the coarseness of a particular type of sand. It takes 2.8 seconds for the sand to empty out of the funnel. Find the flow rate of the sand in milliliters per second. (1 mL = 1 cm³)

**Solution**

STEP 1 Find the volume of the funnel using the formula for the volume of a cone.

$$V = \frac{1}{3}(\pi r^2)h = \frac{1}{3}\pi(4^2)(6) \approx 101 \text{ cm}^3 = 101 \text{ mL}$$

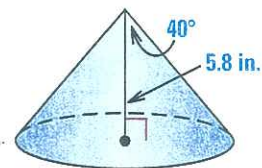
STEP 2 Divide the volume of the funnel by the time it takes the sand to empty out of the funnel.

$$\frac{101 \text{ mL}}{2.8 \text{ s}} \approx 36.07 \text{ mL/s}$$

▶ The flow rate of the sand is about 36.07 milliliters per second.

**GUIDED PRACTICE for Examples 3, 4, and 5**

- Find the volume of the cone at the right. Round your answer to two decimal places.
- A right cylinder with radius 3 centimeters and height 10 centimeters has a right cone on top of it with the same base and height 5 centimeters. Find the volume of the solid. Round your answer to two decimal places.
- WHAT IF?** In Example 5, suppose a different type of sand is used that takes 3.2 seconds to empty out of the funnel. Find its flow rate.



11.7 EXERCISES

HOMEWORK KEY

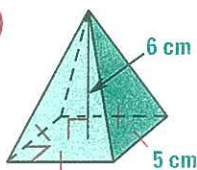
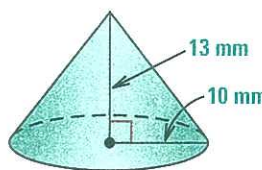
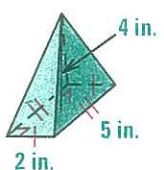
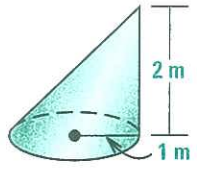
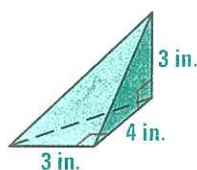
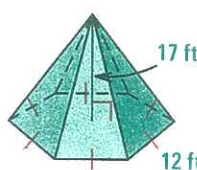
- = See **WORKED-OUT SOLUTIONS**
Exs. 3, 17, and 33
- ★ = **STANDARDIZED TEST PRACTICE**
Exs. 2, 11, 18, and 35
- ◆ = **MULTIPLE REPRESENTATIONS**
Ex. 39

SKILL PRACTICE

- VOCABULARY** Explain the difference between a *triangular prism* and a *triangular pyramid*. Draw an example of each.
- ★ WRITING** Compare the volume of a square pyramid to the volume of a square prism with the same base and height as the pyramid.

EXAMPLE 1 for Exs. 3–11

VOLUME OF A SOLID Find the volume of the solid. Round your answer to two decimal places.

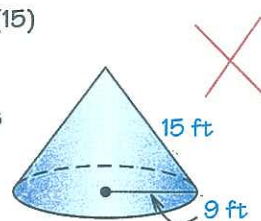
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ERROR ANALYSIS Describe and correct the error in finding the volume of the right cone or pyramid.

9.
$$V = \frac{1}{3}\pi(9^2)(15)$$

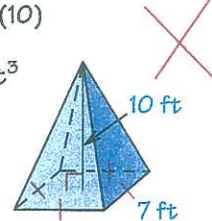
$$= 405\pi$$

$$\approx 1272 \text{ ft}^3$$



10.
$$V = \frac{1}{2}(49)(10)$$

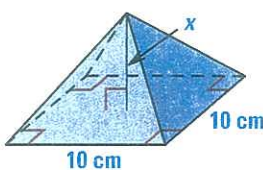
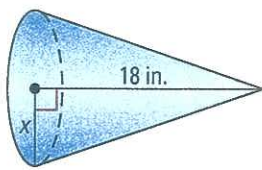
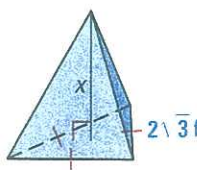
$$= 245 \text{ ft}^3$$



- ★ MULTIPLE CHOICE** The volume of a pyramid is 45 cubic feet and the height is 9 feet. What is the area of the base?
 (A) 3.87 ft^2 (B) 5 ft^2 (C) 10 ft^2 (D) 15 ft^2

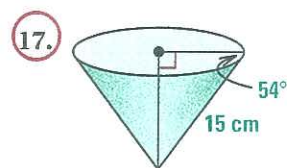
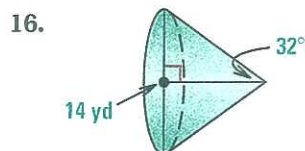
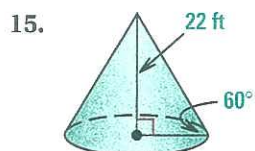
EXAMPLE 2 for Exs. 12–14

xy ALGEBRA Find the value of x .

- Volume = 200 cm^3

- Volume = $216\pi \text{ in.}^3$

- Volume = $7\sqrt{3} \text{ ft}^3$


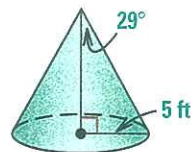
EXAMPLE 3
for Exs. 15–19

VOLUME OF A CONE Find the volume of the right cone. Round your answer to two decimal places.



18. **★ MULTIPLE CHOICE** What is the approximate volume of the cone?

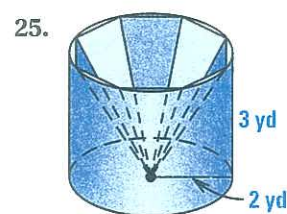
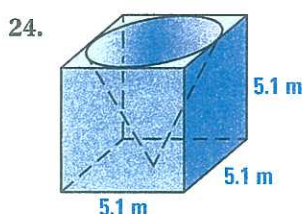
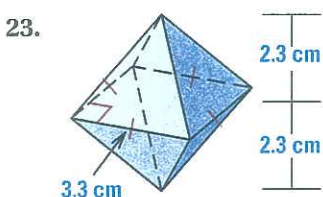
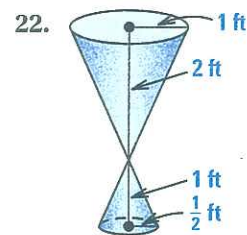
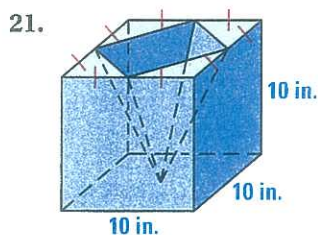
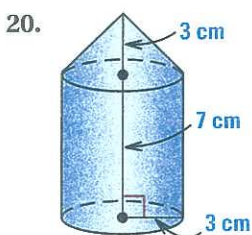
- (A) 47.23 ft^3 (B) 236.15 ft^3
(C) 269.92 ft^3 (D) 354.21 ft^3



19. **HEIGHT OF A CONE** A cone with a diameter of 8 centimeters has volume 143.6 cubic centimeters. Find the height of the cone. Round your answer to two decimal places.

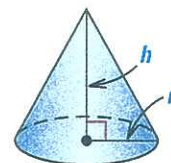
EXAMPLE 4
for Exs. 20–25

COMPOSITE SOLIDS Find the volume of the solid. The prisms, pyramids, and cones are right. Round your answer to two decimal places.



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26. **CHANGING VOLUME** A cone has height h and a base with radius r . You want to change the cone so its volume is doubled. What is the new height if you change only the height? What is the new radius if you change only the radius? *Explain.*



27. **FINDING VOLUME** Sketch a regular square pyramid with base edge length 5 meters inscribed in a cone with height 7 meters. Find the volume of the cone. *Explain* your reasoning.

28. **CHALLENGE** Find the volume of the regular hexagonal pyramid. Round your answer to the nearest hundredth of a cubic foot. In the diagram, $m\angle ABC = 35^\circ$.

